Letter Symbols to Designate Microwave Bands

At a recent meeting of RCRD 16 (Waveguides),[1] members discussed the question of the use of letters to designate microwave wavebands. It was noted that the use of these letters is historical, arising in the first instance from security considerations in wartime radar, and that the letters follow no logical sequence. Moreover, there is no unanimity, either in Great Britain or in the United States, as to the meaning of these letters, and there is much proliferation in some quarters, with consequent confusion. Accordingly, there appeared to be some case for dropping the existing system and either having a logical and systematic designation or, possibly, none at all. However, the members felt, despite this unpromising outlook, that the letters really did serve a useful purpose, and that some terms, such as *X*-band, are too ingrained to be dropped. It was suggested that these terms might be likened to the use of colors to designate parts of the optical spectrum. Although the edges of the band designated "yellow," for example, may not be too clearly defined, nevertheless it is a useful term to have; although in accurate scientific work one would naturally use the appropriate unit, Angstroms, wavenumber, or frequency. Overlap in color designation occurs, of course, and one can use phrases like yellow-green to describe them. Similarly, it was felt that with a suitable, small number of letter terms, phrases like *X*-band would take on a useful meaning, to be supplemented by accurate wavelength or frequency descriptions when appropriate. Terms like *X-J* might be used descriptively to denote overlap regions.

Although no implication of standardization, either national or international, is intended in this letter, the committee members felt it would be of value to go on record with the following agreed list of designations, and to hope that, where possible, individuals would use the letters with these meanings. The list is in substantial accord with existing usage, both in Great Britain and in the United States. The slight overlap between *C*- and *X*-bands is unfortunate, but it was felt that this was essential to maintain continuity with existing use of these letters. The recommended list is as follows.

Band	Approximate Limits (Gc)
L	1– 2
S	2– 4
C	4– 8
X	7–12
J	12–18
K	18–26
Q	26–40
V	40–60
O	60–90

L. Lewin
Standard Telecommun. Labs., Ltd.
Harlow, Essex
England

Manuscript received May 4, 1964.

[1] United Kingdom Radio Component Research and Development Sub-Committee 16 (waveguide components).

Tangent Relation for Determining Immitance Inverter Parameters

The immitance (impedance or admittance) inverter of Cohn[1] and Matthaei[2] can be described by a tangent relation similar to that of Weissfloch.[3] The relationship leads to a simple method of extracting the inversion parameter and intrinsic angular length of a lossless, but otherwise arbitrary, discontinuity.

The immitance-inverter equivalent circuit of a section of waveguide (or transmission line) is defined by the relations illustrated in Fig. 1. Distances d_1 and d_2 are measured to the left and to the right, respectively, of the arbitrary reference plane T_0. No restriction is placed on the sign of d_1 or d_2; either or both may be negative, indicating that the prototype input or output line leading to the inverter circuit is longer than the actual line measured to T_0.

Define

$$\phi_1 = \beta_1(D - d_1) \tag{1}$$

$$\phi_2 = \beta_2(S - d_2) \tag{2}$$

where D and S are measured to the short and minimum positions, as shown in Fig. 2. Applying the transmission-line impedance formula yields

$$Z_a = -jZ_0' \tan \phi_1 \tag{3}$$

$$Z_b = jZ_0'' \tan \phi_2 \tag{4}$$

from which it follows that

$$K^2 = J^{-2} = Z_0'Z_0'' \tan \phi_1 \tan \phi_2 \tag{5}$$

or

$$k^2 = \tan \phi_1 \tan \phi_2 \tag{6}$$

where

$$k = \frac{K}{\sqrt{Z_0'Z_0''}} = \frac{\sqrt{Y_0'Y_0''}}{J}. \tag{7}$$

Eq. (7) defines the normalized inversion factor k. The Weissfloch tangent relation is[4]

$$N^2 = -\tan(\phi_1 + \theta_a) \cot(\phi_2 + \theta_b) \tag{8}$$

where θ_a and θ_b define a different pair of reference planes. It can be shown that

$$N^2 = n^2 \frac{Z_0''}{Z_0'} \tag{9}$$

where n is the turns ratio of an ideal transformer.

The plot of $\beta_1 D$ vs $\beta_2 S$ has the well-known form illustrated in Fig. 3. Points ①, ②, ③, etc., are solutions of (6) that also satisfy the equation

$$\sin 2\phi_1 = \sin 2\phi_2. \tag{10}$$

Manuscript received May 11, 1964.

[1] S. B. Cohn, "Direct-coupled-resonator filters," Proc. IRE, vol. 45, pp. 187–196, February, 1957.

[2] G. L. Matthaei, "Design of wide-band (and narrow-band) band-pass microwave filters on the insertion loss basis," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8 pp. 580–593; November, 1960.

[3] A. Weissflock, "Anwendung des transformersatzes uber verlustlose vierpolen auf die hinter einander schaltung, von vierpolen," *Hochfrequenz. und Elektroak.*, vol. 61, pp. 19–28; January, 1943.

[4] N. Marcuvitz, "On the reproduction and measurement of waveguide discontinuities." Proc. IRE, vol. 36, pp. 728–735; June, 1948.

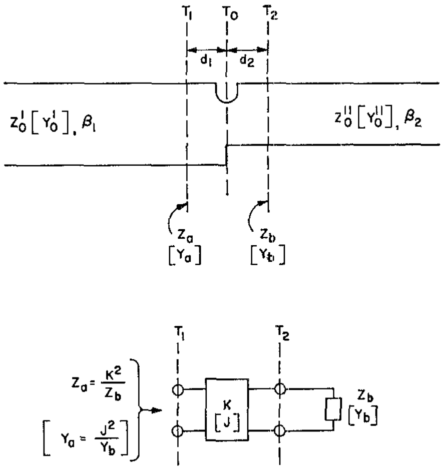

Fig. 1—Generalized waveguide discontinuity and immitance inverter equivalent circuit.

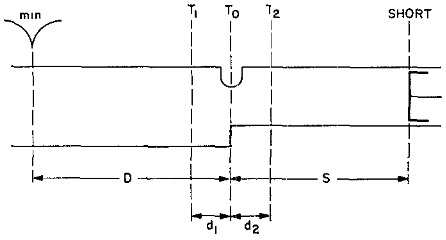

Fig. 2—Definitions of distances D, S, d_1 and d_2.

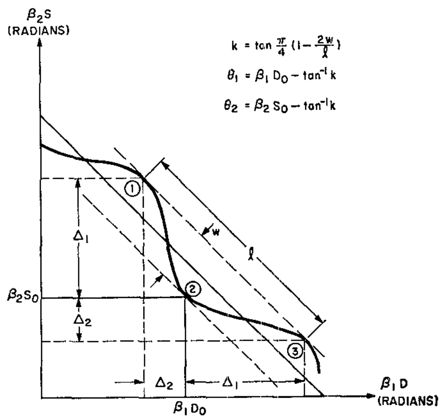

Fig. 3—Tangent relations for inversion constant and intrinsic angles derived from ϕ_1 vs ϕ_2 plot.

One of these points $(\beta_1 D_0, \beta_2 S_0)$ is identified with the relation

$$\phi_1 = \phi_2 = \tan^{-1} k. \tag{11}$$

The others lie at intervals of Δ_1 or Δ_2, where

$$\Delta_1 = \pi - 2 \tan^{-1} k \tag{12}$$

$$\Delta_2 = 2 \tan^{-1} k. \tag{13}$$

By simple trigonometry,

$$w = \frac{\Delta_1 - \Delta_2}{\sqrt{2}} \tag{14}$$

$$l = \pi\sqrt{2} \tag{15}$$

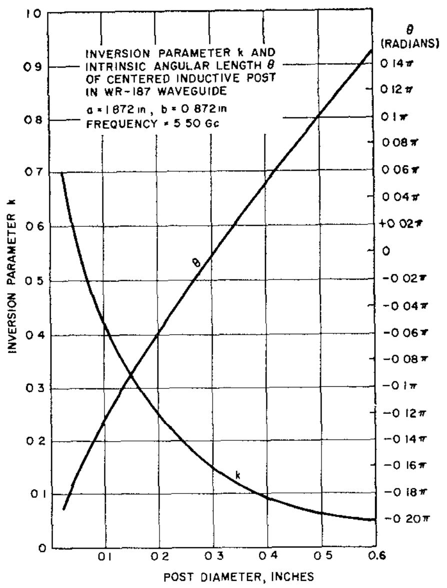

Fig. 4—Parameters of an inductive post in WR-187 waveguide.

where w and l are the amplitude and period shown in Fig. 3 Thus,

$$k = \tan\frac{\pi}{4}\left(1 - \frac{2w}{l}\right). \quad (16)$$

Eq. (16) is a very practical expression, because plotting errors can be averaged out by using the ratio of the average amplitude $\bar{w}$ to the average period $\bar{l}$.

The intrinsic angular lengths, $\theta_1 = \beta_1 d_1$ and $\theta_2 = \beta_2 d_2$, are given by

$$\theta_1 = \beta_1 D_0 - \frac{\pi}{4}\left(1 - \frac{2w}{l}\right) \quad (17)$$

$$\theta_2 = \beta_2 S_0 - \frac{\pi}{4}\left(1 - \frac{2w}{l}\right) \quad (18)$$

where $\beta_1 D_0$ and $\beta_2 S_0$ are the coordinates of an "inside peak," such as point ②. The correct point to use is the one having the most nearly equal values of $\beta_1 D$ and $\beta_2 S$. If the network happens to have bilateral symmetry about plane T_0, $\beta_1 D_0$ and $\beta_2 S_0$ will be exactly equal; in general, they will differ by less than $\pi/2$.

An application for this measurement arises in the design of filters using thick inductive posts. The impedance behavior with frequency of a thick post does not follow that of a narrow iris, and the intrinsic angular length cannot be computed from the handbook value of the shunt susceptance. In Fig. 4 are shown values of θ and k derived from tangent-relation plots of data taken with centered inductive posts in WR-187 waveguide at 5.5 Gc. It is interesting that the sign of θ changes at a diameter of approximately $a/6$. For posts of larger diameter, $d_1 = d_2 = \theta/2\beta$ is positive, and directly-coupled resonator sections (of a cascade-resonator filter) must be slightly longer than $\lambda_{g0}/2$.

D. B. Churchill
Microwave Engineering Dept. D-40
Sperry Gyroscope Co.
Great Neck, N. Y.

Integral Quotient in Measurements of Ambipolar Diffused Plasma with TE_{011} Cavity

Despite some known exact solutions of plasma loaded TM_{010} cavity,[1,2] the TE_{011} mode should be used due to measurement-technical reaons in the case of large electron densities and considerable losses.[3,4] This communication is connected with the mathematical treatment of measurement results in the case of TE_{011} mode and ambipolar diffusion in discharge tube. The treatment is based on Slater's[5] expression for discharge admittance and the assumption of small perturbations, but not on any special electron theoretically derived plasma conductivity formula.

Owing to the ambipolar diffusion in the discharge tube, the distribution of electron density n along the radius r is

$$n = n_0 J_0(2.405 r/r_0) \quad (1)$$

where n_0 is the electron density at the axis and r_0 the inside radius of the discharge tube. The plasma conductivity is proportional to the electron density and obeys the same distribution.

Starting from Slater's discharge admittance,[5] the expressions for the real part σ_{r0} and the imaginary part σ_{i0} of plasma conductivity at the axis can be derived and are

$$\begin{cases} \sigma_{r0} = (g_d \omega_0 \epsilon_0/\beta) Q \\ \sigma_{i0} = (-2\Delta\omega_0 \epsilon_0) Q \end{cases} \quad (2)$$

where g_d is the discharge conductance, ω_0 the angular resonant frequency, $\Delta\omega_0$ the change of ω_0 due to the discharge plasma, ϵ_0 the dielectric constant of free space, β the factor depending on coupling between transmission line and cavity, and Q the quotient of two volume integrals.

In the case of ambipolar diffusion and TE_{011} cavity, after integration with respect to z and ϕ (in cylindrical coordinates)[6] one has

$$Q = Q(R/r_0) = Q_1(R)/Q_2(R/r_0, r_0) = \int_0^R r J_1^2(3.832 r/R) dr \Big/ \int_0^{r_0} r J_0(2.405 r/r_0) J_1^2(3.832 r/R) dr. \quad (3)$$

R is the inner radius of the cavity.

The integration in the numerator of (3) can be performed simply and one has $Q_1(R) = \frac{1}{2} R^2 J_0^2(3.832)$. The values of $Q_2(R/r_0, r_0)$ at $r_0 = 1$ cm have been computed on an electronic digital computer. The results are presented in Table I. Q_2 can be found for other values of r_0 by noting that it is proportional to the square of r_0. The integral quotient Q is only the function of R/r_0 and is presented in Table I and in Fig. 1 for practical values of the argument R/r_0.

TABLE I
Computed Values of Denominator Q_2 at $r_0 = 1$ cm and Integral Quotient Q as Function of R/r_0, the Ratio of Cavity Radius to Discharge Tube Radius

R/r_0	Q_2, cm²	Q
3	0.022358	32.650
4	0.013688	94.805
5	0.0091115	222.54
6	0.0064640	451.71
7	0.0048106	826.14
8	0.0037141	1397.6
9	0.0029514	2225.9
10	0.0024005	3378.8

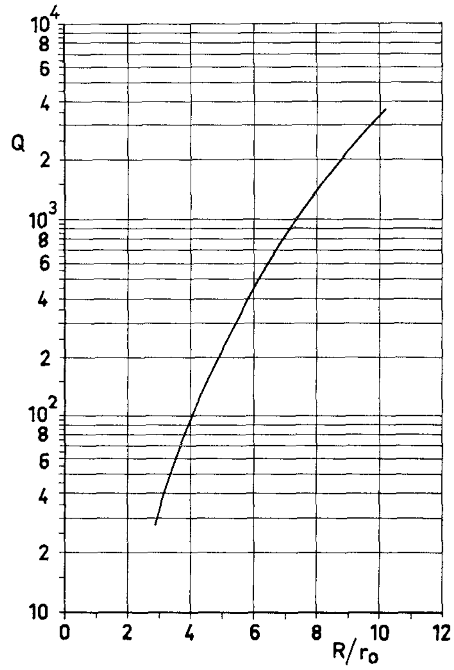

Fig. 1—The quotient Q of two volume integrals in the measurement of ambipolar diffused plasma with TE_{011} cavity. R is the inner radius of cavity, r_0 the inner radius of discharge tube.

Paavo Jääskeläinen
Electronics Div.
Finnish Cable Works Co.
Helsinki, Finland

Manuscript received May 11, 1964.

[1] B. Agdur and B. Enander, "Resonances of a microwave cavity partially filled with a plasma," *J. Appl. Phys.*, vol. 33, pp. 575–581; February, 1963.

[2] P. Hedvall, "Cavity method for measuring plasma properties," *Ericsson Technics*, vol. 19, pp. 97–107; 1963.

[3] K. B. Persson, "Limitations of the microwave cavity method of measuring electron densities in a plasma," *Phys. Rev.*, vol. 106, pp. 191–195; April, 1957.

[4] S. J. Buchsbaum and S. C. Brown, "Microwave measurements of high electron densities," *Phys. Rev.*, vol. 106, pp. 196–199; April, 1957.

[5] J. C. Slater, "Microwave electronics," *Revs. Mod. Phys.*, vol. 18, pp. 441–512; October, 1946.

[6] P. Jaaskeläinen, "On attenuation and electrical length of a plasma loaded helical transmission line," *Acta Polytech. Scand.*, Ph 23; 1963.

On "Status Report on International Millimeter Waveguide Flange Standards"[1]

In his communication, Anderson described the state of international standardization of millimeter waveguide flanges and concluded that in the absence of a suitable

Manuscript received May 14, 1964.

[1] T. N. Anderson, IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 427–429; September, 1963.